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Pronounced Focusing of Hermite Cosh Gaussian Beam for Mode Index $M=0$ in Plasmas

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A B S T R A C T

Self focusing of Hermite Cosh Gaussian (HChG) laser beams in a rippled density plasmas has been observed taken into account of relativistic nonlinearities. Non linear parabolic partial differential equation governing the evolution of complex envelope in slowly varying approximation is solved using paraxial approximation in a periodically modulated density profile. Pronounced self focusing is observed for mode index $m=0$ and for a particular set of decentered parameters.

Introduction

At high intensities, the interaction of laser beam with plasmas have received a great deal of attention due to its relevance in fundamental research as well its technological applications such as charged particle acceleration (Lotov *et al.*, 2014), x-ray source (Arora *et al.*, 2014), inertial confinement fusion (Brian M. Haines, 2015), high harmonic generation (Mendonca *et al.*, 2015), and attosecond pulse generation (Tosa *et al.*, 2012). Interaction of intense laser beam with non linear medium gives rise to non linear phenomenon such as self phase modulation (Schroeder *et al.*, 2011), ponderomotive self focusing (B. Bokaei *et al.*, 2013), self trapping (Aakash and Thomas, 2014) and parametric instabilities (Sukhdeep Kaur and Sharma,

2011). Self focusing of laser beams in a plasmas has a topic of continuous interest from last three decades due to its wide ranging applications. One needs to focus on long propagation distance of intense laser beams in a plasma without loss of energy. Self focusing of laser beams in a plasmas has a topic of continuous interest from last three decades due to its wide ranging applications. Recently emphasis on propagation of various HChG beams in complex optical system and turbulent atmosphere have been investigated by different scientific community (Tang *et al.*, 2006; Bai *et al.*, 2010).

As the laser beam propagates through plasmas, the dielectric constant of the

plasma changes significantly, results in modifications of refractive index via relativistic and ponderomotive nonlinearities. Relativistic self focusing occurs at relativistic velocity ($v \cong c$) when mass of electron increases by a factor $\gamma = 1/\sqrt{1 - \frac{v^2}{c^2}}$. The self focusing is counterbalanced by diffraction effect.

Initial theoretical analysis on self focusing was reported by Akhmanov *et al.*, (1968) and it is further developed by scientific community. Recently Gill *et al.*, (2011) have studied self focusing of Cosh Gaussian laser beam in plasma with weakly relativistic and ponderomotive regime. They have discussed the effect of decentred parameter b on self focusing, self phase modulation and self trapping of Cosh Gaussian beam. Kaur and Sharma (2009) have developed analytical and numerical analysis for propagation of an intense laser pulse in a plasma with a periodically modulated density profile using envelope equations. Overall self focusing length increases and minimum spot size decreases with the wave number of the ripple. Nanda and Nitikant (2014) discussed enhanced relativistic self focusing of Hermite Cosh Gaussian laser beam in the presence of plasma density ramp. Proper selection of decentred parameter results in strong self focusing of HChG beam. Patil *et al.*, (2010) have investigated the focusing of HChG laser beams in magneto plasma by considering ponderomotive nonlinearity. The effect of mode index and decentred parameter on the self focusing of the beams has been discussed. Additional self focusing is observed for higher decentred parameter but ordinary mode of collision less magneto plasma is less susceptible for self focusing than extra ordinary mode.

This paper presents an investigation of pronounced focusing of HChG laser beams in rippled density plasmas taking into account of relativistic nonlinearities. Density ripple gives impact on propagation of laser beam in a plasma. Non linear parabolic partial differential equation governing the evolution of complex envelope in slowly varying envelope approximation is solved using paraxial approximation in a periodically modulated density profile. The paper is as follow, section II represents mathematical formulational of self focusing of the laser beam. Results and discussion are incorporated in section III followed by references.

Self focusing of the beam

In the absence of external charge and current, Ampere's and Faraday's laws can be written as

$$\vec{\nabla} \times \vec{B} = \frac{1}{c} \frac{\partial \vec{D}}{\partial t} \tag{1}$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} \tag{2}$$

where \vec{E} and \vec{B} are the electric and magnetic field vectors, \vec{D} is displacement current given as $\vec{D} = \epsilon \vec{E}$ and ϵ is dielectric permittivity in which induced current density due to laser plasma interaction exists. Taking curl of equation (2) and using equation (1) we get,

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\frac{1}{c^2} \frac{\partial^2 \vec{D}}{\partial t^2}$$

By solving above equation we obtain the general wave equation of laser beam propagation is,

$$\nabla^2 E - \frac{\omega^2}{c^2} \varepsilon E + \nabla \left(\frac{E \cdot \nabla(\varepsilon)}{\varepsilon} \right) = 0 \quad (3)$$

For $(1/k^2) \nabla^2(\ln \varepsilon) \ll 1$, we get

$$\frac{\partial^2 E}{\partial z^2} + \frac{\partial^2 E}{\partial r^2} + \frac{1}{r} \frac{\partial E}{\partial r} + \varepsilon \frac{\omega^2}{c^2} E = 0 \quad (4)$$

The solution of equation (4) is of the form,

$$E = A(r, z) e^{i(\omega t - kz)} \quad (5)$$

The field distribution of HChG laser beam propagating in a plasma is given by

$$E(r, z) = \frac{E_0}{f(z)} \left[H_m \left(\frac{\sqrt{2}r}{r_0 f(z)} \right) \right] e^{\frac{b^2}{4}} \left\{ e^{-\left(\frac{r}{r_0 f} + \frac{b}{2} \right)^2} + e^{-\left(\frac{r}{r_0 f} - \frac{b}{2} \right)^2} \right\} \quad (6)$$

where m is the mode index associated with the Hermite polynomial H_m , E_0 is the amplitude of HChG beam for central position at $r = z = 0$, r_0 is the initial spot size of the laser beam, b is the decentred parameter, $f(z)$ is the dimensionless beam width parameter of the laser beam.

As the laser beam propagates through the plasma, it imparts oscillatory velocity to the electrons given by $v = \frac{eE}{m_0 \omega c}$, where $\gamma = \sqrt{1 + \alpha EE^*}$ is the intensity dependent relativistic factor with $\alpha = \frac{e^2}{m_0^2 \omega^2 c^2}$, here m_0 , ω , e are the rest mass of electrons, angular frequency of incident beam, charge on electron and c is the speed of light in vacuum.

The dielectric constant for non linear medium is given by

$$\varepsilon = \varepsilon_0 + \varphi(EE^*) \quad (7)$$

Following Kaur et al (2009), the equilibrium electron density n be sinusoidal,

$$n = n_0(1 + \alpha_2 \cos qz), \quad (8)$$

where n_0 is the maximum electron density and q is the ripple wave number.

In the absence of density transition, considering only the relativistic mass of the electrons, $m_e = m_0 \gamma$. In the presence of density ripple, $n = n_0(1 + \alpha_2 \cos qz)$, substituting n in dielectric constant of the plasma gives the non linear part as follows,

$$\varphi(EE^*) = \left[\frac{\omega_{p0}^2}{\gamma \omega^2} + \frac{\omega_{p0}^2}{\gamma \omega^2} \alpha_2 \cos qz \right] \left[1 - \frac{1}{(1 + \alpha EE^*)^{1/2}} \right] \quad (9)$$

The complex amplitude $A(r, z)$ may be expressed as,

$$E(r, z) = E_0(r, z) e^{-ik(z)S(r, z)} \quad (10)$$

where E_0 and S are the real functions of r and z . Following Sukhdeep and Sharma, (2009) for eikonal S in Eq. (10). Now substituting (5) and (10) in equation (3) and neglecting $\frac{\partial^2 A}{\partial z^2}$, we get a complex differential equation with real and imaginary parts. Substituting A_0^2 in real and imaginary parts, where A_0^2 is given by,

$$A_0^2 = \frac{E_0^2}{f^2(z)} \left[H_m \left(\frac{\sqrt{2}r}{r_0 f} \right) \right]^2 e^{\frac{b^2}{2}} \left\{ e^{-2\left(\frac{r}{r_0 f(z)} + \frac{b}{2} \right)^2} - e^{-2\left(\frac{r}{r_0 f(z)} - \frac{b}{2} \right)^2} + 2e^{-\left(\frac{2r^2}{r_0^2 f^2(z)} + \frac{b^2}{2} \right)} \right\} \quad (11)$$

Using the values of equation (10) and (11) in real part equation, we get the equation governing the evolution of beam width

parameter. Following Nitikant et al (2014), the equation of beam width parameter for m=0 mode index in the presence of ripple density is given by (12),

$$\left[1 + \frac{\xi \alpha_2 q' \sin(q' \xi) \frac{\omega_{p0}^2}{\gamma \omega^2}}{2 \left(1 - \frac{\omega_{p0}^2}{\gamma \omega^2} - \frac{\omega_{p0}^2}{\gamma \omega^2} \alpha_2 \cos(q' \xi) \right)} \right] \frac{d^2 f}{d\xi^2} - \frac{(4-4b^2)}{f^3} + \frac{4\alpha E_0^2}{f^3} \left(\frac{\omega_{p0}^2}{\omega^2} + \frac{\omega_{p0}^2}{\omega^2} \alpha_2 \cos(q' \xi) \right) \left(\frac{\omega r_0}{c} \right)^2 \left(1 + \frac{4\alpha E_0^2}{f^2} \right)^{-3/2} e^{\frac{b^2}{2}} = 0 \tag{12}$$

Here $q' = qR_d$, $\xi = \frac{z}{R_d}$ is the normalized propagation distance, R_d is the diffraction length. Using boundary conditions, at $\xi = 0$, $f = 1$, and $df/d\xi = 0$.

Results and Discussion

We have solved Eq. (12) numerically taking into account of intensity of laser beam is $I = 1.38 \times 10^{16} \text{ W/cm}^2$, and the frequency of incident laser beam is $\omega = 1.778 \times 10^{15} \text{ rad/sec}$. We have plotted variation of beam width parameter f as a function of normalized distance of propagation ξ for different parameters. Fig 1 shows variation of beam width parameter with normalized distance of propagation at m=0 mode of HChG beam in the presence of density ripple. The other parameters are $r_0\omega/c = 500$, $b = 0.9$, and ripple wave number $q_1 = 13, 14, 50$. The depth of density modulation is $\alpha_2 = 0.2$, $\alpha E_0^2 = 0.1$,

$\omega_{p0}/\omega = 0.75$, and $r_0 = 84.36 \mu\text{m}$. We have taken $\lambda_L = 1 \mu\text{m}$. We found that while increasing the ripple wave no, the beam width parameter self focused and shows periodic focusing.

At normalized ripple wave no $q_1 = 50$ the beam width parameter decreases initially but diffraction effect predominates. As the beam propagates deep into the plasma, its beam converges towards lower value of normalized propagation distance followed by periodic focusing and de-focusing. The effect of density ripple is to cause overall increase in the self focusing length.

Fig 2 represents the variation of beam width parameter with normalized distance of propagation at m=0 with density ripple for different value of decentred parameter/ The other parameters are $r_0\omega/c = 500$, ripple wave number $q_1 = 30$, and $b = 0, 0.9, 1.8$. We found that as we increases the decentred parameter at fixed wave no of density ripple $q_1 = 30$, strong self focusing occurs at $\xi = 0.0020$.

It is clear from the figure that while increasing the decentred parameter, self focusing of HChG beams shifted towards lower value of normalized propagation distance and periodic focusing and de-focusing is seen due to predominance of diffraction effect. In conclusion we found that ripple wave no and decentred parameter can improve focusing length which has got practical applications in inertial fusion energy experiments. One can solve it further for ponderomotive nonlinearity.

Figure.1 Variation of Beam Width Parameter with Normalized Distance of Propagation at $m=0$ with Density Ripple. The Other Parameters are and Ripple Wave Number

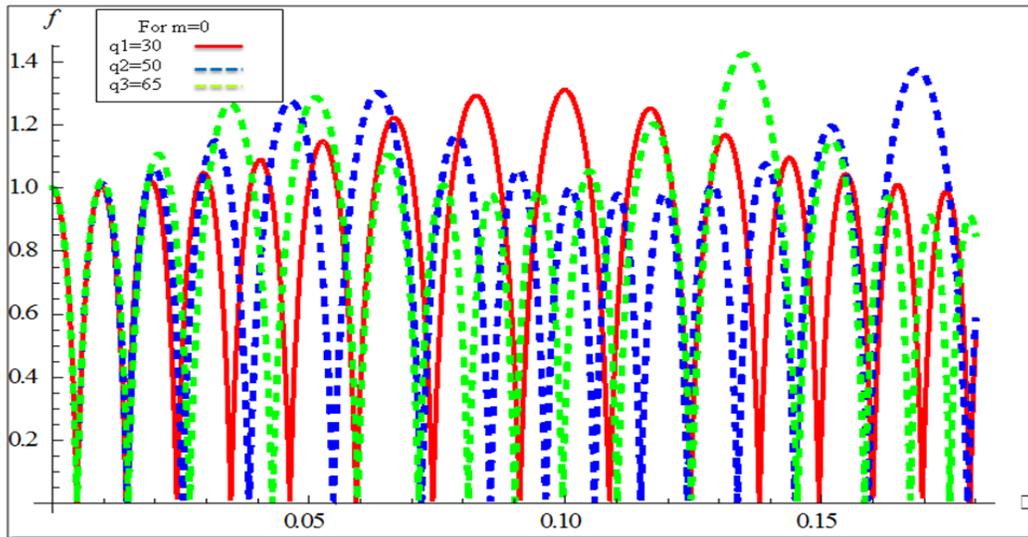
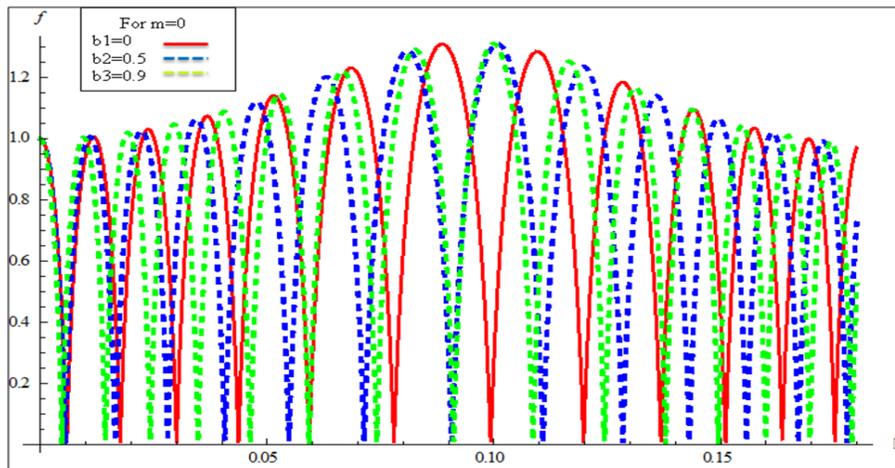


Figure 2: Variation of beam width parameter with normalized distance of propagation at $m=0$ with density ripple. The other parameters are ripple wave number , and



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